Balance-Of-Payments Constrained Growth: a Post-Keynesian Model with Capital Inflows

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Balance-Of-Payments Constrained Growth: a Post-Keynesian Model with Capital Inflows

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RESUMO
Neste trabalho propomos uma breve análise teórica sobre a relação entre mobilidade internacional de capitais e crescimento econômico, aqui observada sob a perspectiva do crescimento liderado pela demanda. O trabalho está organizado em quatro seções além da introdução. Inicialmente, evocamos brevemente as proposições centrais que constituem a concepção de crescimento liderado pela demanda. Na seção seguinte, o modelo de crescimento com restrição no balanço de pagamentos proposto por Thirlwall (1979) é revisto e desenvolvimentos posteriores, que buscam incorporar o papel dos fluxos de capital nesse arcabouço analítico, são analisados. Com base nesses elementos desenvolvemos, em seguida, um modelo teórico com vistas a estudar a relação entre crescimento com restrição externa e controle de capitais, formalmente introduzido como uma variável de política macroeconômica. Na parte final do ensaio, são traçadas considerações finais sobre o modelo. Palavras-chave: crescimento econômico, restrição externa, controle de capitais

ABSTRACT
This paper proposes a brief theoretical analysis of the relationship between international capital mobility and economic growth, seen in the perspective of demand-led growth. The work comprises four sections, besides the introduction. First of all, we briefly present the central propositions that constitute the so-called demand-led growth perspective. We then review the balance-of-payments constrained growth model, as originally proposed by Thirlwall (1979), as well as subsequent efforts to incorporate capital flows into this analytical framework. Building on this theoretical structure, a model is proposed in order to study the connections between balance-of-payments constrained growth and capital controls, which is formally introduced as a macroeconomic policy variable. In the last section, we present the final remarks on this model.

Key-words: economic growth, balance-of-payments constraint, capital controls.
1. INTRODUCTION

In this paper we present a brief theoretical analysis of the relationship between international capital mobility and economic growth, seen from the perspective of demand-led growth. The principle of effective demand, as originally proposed by Keynes (1936) in his General Theory, considers economic growth to be induced by the temporal behaviour of aggregate demand. This does not mean that the supply-side is irrelevant, but rather that demand is the dominant force in the process. In open economies, a fundamental constraint to the sustained expansion of income is the equilibrium in the balance-of-payments. The balance-of-payments constraint is of particular relevance for developing countries, since external imbalances are not automatically corrected via relative prices. In face of the growing international financial liberalisation of the last decades, it is imperative to understand the role played by capital flows in this context. Specifically, it is necessary to question whether the access to foreign capital can, in fact, contribute to overcoming the balance-of-payments constraint of less developed economies. Likewise, it is necessary to investigate to what extent controlling these flows can be desirable for long-term growth. We approach these questions based on the post-Keynesian/Kaldorian literature, seminally developed by Thirlwall during the end of the 1970s.

This paper comprises four sections, besides this introduction. First, we briefly present the main propositions of the demand-led growth perspective. Next, we review the balance-of-payments constrained growth model proposed by Thirlwall (1979) and its later developments, which seek to include the role of capital flows in the analytical framework. Based on these elements, we then develop a simplified theoretical model in order to study the relationship between balance-of-payments restrained growth and the control of the net capital inflows, formally introduced as a macroeconomic policy variable. This analysis distinguishes between different kinds of capital flows, and seeks to establish a clear association between flows and stocks when investigating the relationship between economic growth and foreign debt accumulation. Due to the complexity of the model, we formally combine elements of analytical and numerical resolution, employing computer simulations. In the last section of the essay, we present some concluding remarks on the model.

2. DEMAND-LED GROWTH

The cornerstone of the theory of demand-led growth is a challenge to two central postulates of traditional macroeconomic analysis (SETTERFIELD, 2002). The first establishes that variations in aggregate demand exert but a transitory effect on the utilisation of the productive resources of the economy – basically, due to the existence of incomplete nominal adjustments in the short-term. As the eventual imperfections are eliminated, the economy would thus inevitably converge to equilibrium, the latter being determined by supply-side conditions with full employment of factors of production. The second postulate supposes that the expansion of factors of production, over time, does not depend on the behaviour of demand. These elements together give rise to a certain conception of the growth process, considered to be supply-induced, and ascribes a trajectory for the potential product that continually acts as a powerful gravitational centre, to which the effective product is systematically attracted in the long-term, regardless of the behaviour of aggregate demand.

2 For a critical assessment of this adjustment mechanism, see Dutt and Ros (2007).
Opposed to this theoretical perspective, demand-led growth models assume, first, that at each and every moment the utilisation of the productive resources of the economy is fundamentally determined by aggregate demand. There is, therefore, no such thing as a natural rate of unemployment, or any supply-side-determined equilibrium to which the level of activity is inevitably and inexorably attracted in the long-term. As [Keynes (1996)] showed long ago, equilibria with less than full employment of the productive resources, due to a lack of effective demand, are a real possibility in market economies, even if prices and wages can freely adjust. In this context, the mere expansion of the supply of factors of production (or the increase of their productivity) is not a sufficient condition for guaranteeing the growth of output – these resources can remain idle or be under-utilised if demand conditions thus determine. Hence, the path of the effective product is continuously defined by the historical sequence of short-term results associated to the actual utilisation of the available resources, which is determined by demand.

The other central aspect of this analysis regards the endogenous character of the potential product of the economy. More precisely, in this theoretical framework the very evolution of the productive resources over time is significantly influenced by the growth of demand. According to McCombie and Thirlwall, "Most resources for growth, such as the quantity and quality of labour inputs, capital accumulation and improved productivity through technical progress, are elastic in supply and endogenous to an economic system, dependent on the growth of output itself" (p. 1, 2004). It can thus be said that the path of the potential product defines, at all times, an upper limit for the level of economic activity – a limit which is itself sensitive to current output. In this sense, demand creates its own supply, within certain limits. Long-term growth is thus fundamentally driven by the behaviour of aggregate demand.

According to this view, many mechanisms explain the endogeneity of the potential product. Firstly, the capital stock desired or considered adequate by firms, as a whole, is crucially dependent on their expectations of growth of the demand for their products. Rapid economic growth tends to create favourable perspectives regarding investments, thus stimulating capital accumulation in the economy and thereby further growth – the so-called accelerator mechanism. The growth of demand also exerts a positive effect on the growth of the labour force, in three main ways. First, it leads to higher participation rates, especially amongst young people, married women and the elderly (THIRLWALL, 2002). Second, it increases overtime work. Third, it promotes the migration of workers, which is to say that booming labour markets tend to attract workers from other regions or countries, increasing the local labour supply. Last, many studies indicate a strong causal relationship between the growth of output (as an independent variable) and the growth of factor productivity. This is associated to the existence of dynamic and static increasing returns to scale, as proposed by Verdoorn's Law. The increasing returns to scale are the result of the interaction of various processes, particularly the learning-by-doing mechanism, the technological progress embodied in capital accumulation and the stimulus to innovation activities produced by the growth of demand (LEÓN-LEDESMA; THIRLWALL, 2002).

According to this analysis, growth differs between countries, essentially, because the growth of demand in these economies follows different pace over time. A question of central importance in this context is, therefore, explaining this divergence. The most plausible explanation highlights that the growth of demand in the economy is subject to various

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3 See Jones (1979).
4 See Dixon and Thirlwall (1975) and McCombie et al. (2003).
institutional and economic constraints. In open economies, an important economic constraint to the expansion of demand is associated to equilibrium in the balance-of-payments.

3. THE BALANCE-OF-PAYMENTS CONSTRAINT

In the neoclassical conception, external equilibrium is not an obstacle to the growth of a country's income and output. This perspective is based, ultimately, on the hypothesis that in the long-term the equilibrium of the balance-of-payments is automatically reached via the adjustment of the real exchange rate. The movement of relative prices would thus guarantee that eventual deficits in current account, associated to the expansion of economic activity, are automatically eliminated via currency devaluation. The external solvency of the domestic economic would thus be assured, with no harm to its long-term growth.

Harrod (1933) questioned the effectiveness of the price system as the mechanism for adjusting the value of a country's exports and imports. He believed that domestic income, and not the exchange rate, was the main variable guaranteeing equilibrium, through the so-called foreign trade multiplier mechanism. In Harrod's model, relative prices remain constant. Starting from an initial situation of equilibrium in the balance of trade, an autonomous increase of imports, for example, initially leads to a foreign deficit. The latter is then corrected via a decrease of domestic activity in the exact amount needed to restore equilibrium. The magnitude of the necessary decrease is determined by the multiplier, which in Harrod's analysis equals the inverse of the marginal propensity to import of the economy. Kaldor revived Harrod's multiplier theory in the mid-1970s. The analysis was, however, still essentially static – i.e., it was developed basically as an exercise of comparative statics employing various equilibrium values for the income level. Thirlwall seminally explored its application to growth theory, in a paper originally published in 1979.

3.1. Thirlwall's model

According to Thirlwall (1979), the dominant constraint to growth in an open economy is its balance-of-payments. His model is based on the proposition that trade deficits are not automatically corrected through the exchange rate, and they cannot be indefinitely financed. The conclusion, then, is that the insufficiency of foreign currency establishes a ceiling for the growth rate of aggregate demand, and, hence, for long-term economic growth.

In Thirlwall’s model, the long-term balance-of-payments equilibrium is, by definition, equal to current account equilibrium. For an economy experiencing balanced growth, this equilibrium requires that the rate of growth of imports be equal to that of the exports of goods and services. Assuming that the real exchange rate is constant in the long-term, the growth of domestic income adjusts in order to assure the solvency of payments in foreign currency.

The balance-of-payments equilibrium, measured in domestic currency, may be described by the following expression:

$$P_{d t} X_t = P_{f t} M_t E_t$$  \hspace{1cm} (1)

---

5 The principle of Harrod's multiplier is essentially analogous to the investment multiplier principle proposed by Keynes in his General Theory.

6 See Kaldor (1975).
where \( P_{dt} \) is the domestic price level; \( X_t \) is the quantity of exports; \( P_{ft} \) is the foreign price level; \( M_t \) is the quantity of imports; and \( E_t \) is the nominal exchange rate, defined as the price of the foreign currency in domestic currency units. Dynamically, the equilibrium condition is thus defined as:

\[
P_{dt} + x_t = P_f + m_t + e_t \quad (2)
\]

where the lower case letters denote the exponential growth rates of the respective variables. The domestic demand for imports is then specified as a multiplicative function; its arguments are the price of imports, the price of domestic goods and the domestic income:

\[
M_t = a \left( \frac{P_f E_t}{P_{dt}} \right)^{\psi} Y_t^{\beta} \quad (3)
\]

In this equation, \( a \) is a constant and \( \psi \) (\(< 0\)) and \( \beta \) (\(>0\)) are, respectively, the price elasticity and the income elasticity of the demand for imports. The expression in parenthesis is the current real exchange rate. Similarly, the demand for the country's exports is also defined as a multiplicative function, depending on relative prices and on the world income:

\[
X_t = b \left( \frac{P_{dt}}{P_f E_t} \right)^{\eta} Y_t^{\delta} \quad (4)
\]

where \( b \) is a constant, \( \eta < 0 \) is the price elasticity and \( \delta > 0 \) the income elasticity of the demand for exports. Consequently, the growth rate of domestic imports and exports can be described as:

\[
m_t = \psi (P_f + e_t - P_{dt}) + \beta Y_t \quad (5)
\]

\[
x_t = \eta (P_{dt} - P_f - e_t) + \delta Y_f \quad (6)
\]

The price and income elasticities of exports and imports are parameters of the model. They reflect the production structure of the economy, as well as the specialisation pattern prevailing in world trade, as a result of the international division of labour.

Substituting equations (5) and (6) in equation (2), the resulting expression can be solved in order to find the growth rate of the economy compatible with the balance-of-payments equilibrium over time:

\[
y_{Bt} = \frac{(1 + \eta + \psi)(P_{dt} - P_f - e_t) + \delta Y_f}{\beta} \quad (7)
\]

If, hypothetically, the real terms of trade prevailing in international trade do not vary, equation (7) assumes the following form.\(^7\)

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\(^7\) According to Thirlwall (1979, p. 50), there are theoretical and empirical evidence supporting the idea that, in the long-term, the real exchange rate tends to be relatively constant, due to arbitrage in the international markets (the law of one price) or to the nominal exchange rate pass-through effect on domestic prices.
According to this expression, the growth rate compatible with the long-term external balance of an economy is determined by the ratio between the growth rate of its exports and the income elasticity of its demand for imports. This result is known in the literature as **Thirlwall's Law**. Thirlwall says it is possible for an economy to grow during long periods of time below its balance-of-payments equilibrium rate, thus accumulating trade surpluses, while the inverse situation is clearly not plausible. Assuming a constant real exchange rate, Thirlwall's Law thus defines, as a rule, a ceiling to which the long-term growth of open economies tends to converge.

Thirlwall’s model presents two main contributions to understanding the long-term growth of economies integrated into world trade. First, assuming a constant real exchange rate, it highlights that foreign disequilibria are not automatically eliminated over time through changes in relative prices. Hence, the fundamental adjustment variable, in a dynamic perspective, is the growth of income. In this sense, it is possible to see the result described by equation (8) as a dynamic version of Harrod's foreign trade multiplier.\(^8\)

The other key contribution of this approach is the vital importance conferred to the structural aspects determining the nature of the country's imports and exports. In particular, if the production structure of an economy and its role in international trade lead to a combination of high income elasticity of the demand for imports and low income elasticity of the demand for the goods it exports, its long-term growth tends to be lower than that of the world economy. As we shall see below, this is an important aspect to be taken into account when analysing the possible contribution of capital flows for overcoming the balance-of-payments constraint of emerging economies.

### 3.2. Capital flows

In Thirlwall's seminal model (1979) the long-term balance-of-payments equilibrium is defined as the equilibrium of the exports and imports of an economy. In a paper published shortly thereafter, Thirlwall and Hussain (1982) claim that developing countries are, frequently, capable of incurring growing deficits in their current account, financed by foreign capital inflows. Hence, they would be able to grow, for extended periods of time, above the level predicted by Thirlwall's Law:

> “It must be recognised, though, that developing countries are often able to build up ever-growing current account deficits financed by capital inflows (which are then written off!) which allow these countries to grow permanently faster than otherwise would be the case. If this is so, growth becomes constrained ultimately by the rate of growth of capital inflows” (Thirlwall; Hussain, 1982, p. 500-501).

In order to deal with the experience of developing economies, Thirlwall and Hussain propose an extension of the original model of balance-of-payments constrained growth. They introduce international capital flows as an alternative source for financing the expenses of the

\[ y_{Bt} = \frac{\delta y_{ft}}{\beta} = \frac{x_t}{\beta} \]  

\(^8\) See, amongst others, McCombie and Thirlwall (1994).
domestic economy in foreign currency. The balance-of-payments equilibrium condition in a
given period of time (in domestic currency units) is then defined as:

\[ P_{dt} X_t + C_t = P_{ft} M_t E_t \]  \hspace{1cm} (9)

where \( C_t \) corresponds to the value of foreign capital inflows, measured in terms of domestic
currency. This equation shows that external balance requires that the expenses with the
imports of goods and services be paid for by the revenues of exports and/or by foreign
financial resources. Dynamically, equilibrium requires the growth rate of currency receipts to
equal the growth rate of expenses in foreign currency:

\[ \left( \frac{E}{R} \right) (p_{dt} + x_t) + \left( \frac{C}{R} \right) (c_t) = p_{ft} + m_t + e_t \]  \hspace{1cm} (10)

where \( E/R \) and \( C/R \) stand for, respectively, the share of exports and of capital flows in total
revenue. These shares are assumed constant in the long-term. Employing the previously used
functional forms for the demand for exports and the domestic demand for imports, as well as
the hypothesis of a constant real exchange rate, Thirlwall and Hussain obtain a modified
version of Thirlwall's Law, now in the context of a model with capital flows:

\[ y_B^* = \frac{(E/R)(x_t) + (C/R)(c_t - p_{dt})}{\beta} \]  \hspace{1cm} (11)

The growth of the economy consistent with the balance-of-payments equilibrium is, therefore,
defined by the ratio between, in the numerator, a weighted sum of the growth of exports and
of capital inflows (in real terms) and, as the denominator, the income elasticity of the demand
for imports. Generally speaking, the equilibrium growth rate is that which guarantees that the
intertemporal sum of the balance-of-payments debits and credits equals zero. A central result
of this model is that the access to international capital may enable a long-term economic
growth superior to that originally predicted by Thirlwall's model (1979).

This result requires the growth rate of real capital inflows to be higher than the growth of
exports, i.e., \( (c_t - p_{dt}) \geq x_t \). However, Thirlwall and Hussein's analysis (1982) leaves behind an
important aspect of the problem – a growing absorption of capital leads to the accumulation
of a stock of foreign debt, which must be remunerated, and its trajectory over time can impose
serious risks to the long-term growth of an economy.

In this sense, Moreno-Brid (1998) observes that although the Thirlwall-Hussain model
introduces the possibility of net inflows of foreign capital, it does not effectively impose any
restrictions to their trajectory. It merely defines the condition that the debit and credit items of
a country's balance-of-payments cancel each other out. According to Moreno-Brid, this
accounting restriction is, in itself, insufficient; it does guarantee the necessary temporal
consistency of flows and stocks that would generate a sustainable foreign debt pattern in the
long run. In order to solve this deficiency, he proposes a modification to the model, redefining
the long-term balance-of-payments equilibrium as the maintenance of a constant ratio of the

\[^9\) As can be easily seen in equation (11).\]
current account deficit relative to the domestic income. Keeping the assumption of a constant real exchange rate, the new model defines the equilibrium growth rate as:

\[ y_{ca} = \frac{\theta x}{\beta - (1 - \theta)} \]  

where \( \theta \) is the initial ratio of the value of exports relative to the value of the imports of goods and services. As noted by the author himself, the Thirlwall-Hussain model could have reached this result if it imposed the condition that the growth of capital inflows was equal to that of income. According to Moreno-Brid, this condition is sufficient for guaranteeing, in the scope of the model, that the foreign debt level of the economy remains asymptotically constant.

Notwithstanding the importance of Moreno-Brid's analysis for understanding the balance-of-payments constraint to growth, Barbosa-Filho (2001) notes that the model has two important limitations. First, its equilibrium growth rate is not necessarily stable. More precisely, stability is restricted to the case in which the income elasticity of the demand for imports is one, an unlikely situation for small open economies. The potential instability of the system is due to the fact that the ratio of exports to imports, formerly considered a parameter, is actually a variable which itself depends on the growth rate of the economy. The second limitation refers to the absence of a necessary distinction, when analysing the build-up of foreign debt, between the imports of goods and non-factor services from interest payments. With this separation, the restriction Moreno-Brid introduces becomes a necessary but insufficient condition for guaranteeing a non-explosive behaviour of foreign indebtedness. Considering these elements, Barbosa-Filho develops a model in which the real exchange rate, as well as the growth rate of income, is an adjustment variable of the foreign accounts. In this analysis, the government conduces the macroeconomic policy (in its monetary and fiscal aspects) so as to assure that the trajectories of the exchange rate and of the income are consistent with the trade balance necessary for maintaining the stability of foreign indebtedness at the level allowed by international credit markets.

This paper aims to contribute to this debate with an alternative analysis of the relationship between growth and foreign debt. We establish a preliminary distinction between the different kinds of capital flows and introduce the control of capital inflows as a macroeconomic policy variable.

4. ECONOMIC GROWTH, FOREIGN DEBT AND CAPITAL CONTROLS

Let the following condition for equilibrium in the balance-of-payments of an economy in a given moment of time, measured in foreign currency:

\[ \frac{P_{d}}{E} X + F = P_{f} M + R \]  

where \( F \) is the net value of foreign capital inflows and \( R \) is the value of interest payments on the existing stock of foreign debt. This specification thus distinguishes between interest payments and the other various imports of goods and services. The demand for these imports

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10 The proposed modification is based on the perception that the long-term balance-of-payments equilibrium does not necessarily require the current account to be balanced. Actually, according to this conception the current account can display a growing deficit, as long as it occurs in tandem with the growth of income.
and the demand for exports are described in the same multiplicative forms considered in the basic balance-of-payments constrained growth model, as in equations (3) and (4). By definition, interest payments depend on the stock of debt and on the interest rate. We consider that this equilibrium condition is continuously observed, regardless of eventual variations in the foreign currency reserves of the economy. This assumption can be justified based on the perception that reserves are, as a rule, limited, and the equilibrium of foreign currency inflows and outflows is a central condition for the stability of the nominal exchange rate.\(^\text{11}\)

Consider two different kinds of capital flows, called direct investment and speculative investment. Direct investment aims at producing goods and services, and does not directly affect the foreign debt of the economy, whereas speculative investment aims strictly at obtaining financial revenue via interest, thus directly impacting the debt stock. Given this distinction, we can decompose the variable \(F\) in the following manner:

\[
F = I_d + I_e
\]

In this expression, \(I_d\) is the net value of capital inflows as direct investment, while \(I_e\) is the net inflow of speculative investment. Based on these concepts, we assume the following differential equation to describe the evolution of foreign debt over time:

\[
\frac{dD}{dt} = rD + I_e - R
\]

The first term in the right side of the equation is the value to be paid as interest on the debt, where \(r\) is the international interest rate. If this value is higher than what is actually paid, \(R\), the difference leads to an increase of the foreign debt of the economy. With these basic elements defined, we shall now analyse three different theoretical situations dealing with distinct hypotheses regarding the behaviour of capital flows and interest payments.

In the first context, called case (A), there is a previously accumulated stock of foreign debt, but the capital inflows are zero and the due interests are not paid, which leads to a cumulative growth of the debt stock. This case, albeit extreme, can be conceived as an approximate (stylised) description of a real situation for an emerging economy that, in a previous scenario of favourable international liquidity, accumulated a debt stock, and after a certain moment begins to face serious difficulties in attracting foreign capital and is incapable of servicing its debt.\(^\text{12}\) In case (B), the economy receives a growing inflow of speculative investment, in line with the growth of the world economy. In this case, the interest is entirely paid, but there is no direct investment. Finally, case (C) describes a situation of nonzero direct investment, maintaining the other elements of case (B). When analysing cases (B) and (C), we shall later introduce the possibility of capital controls, so as to observe its effects on the long-term performance of the economy. When needed, we shall use computer simulations as a complement to the analytical solution of the model.

**CASE (A)**

Consider \(I_e = 0; I_d = 0; R=0\). Given these conditions and substituting equation (3) and (4) in equation (13), we have:

\[11\] This observation is particularly important in countries under inflation targeting regimes with a strong exchange-rate component. See, for example, the post-1999 Brazilian experience. See Giavazzi et al. (2005).

\[12\] See the experience of the Latin-American countries during the 1980s.
After algebraic manipulations, we solve the equation above for the level of domestic income:

$$Y = \left[ \frac{b \left( \frac{P_d}{P_f E} \right)^{1+\eta+\psi}}{a \left( \frac{P_f E}{P_d} \right)} \right]^{\frac{1}{\beta}} Y_f^{\delta/\beta} \tag{17}$$

Assuming that the real exchange rate is constant over time (as well as the price and income elasticities of the model), the growth rate of the economy is the one given by Thirlwall's Law:

$$g = \frac{\delta}{\beta} g_f \tag{18}$$

where $g_f$ is the growth rate of the world economy. However, we shall simultaneously have an explosive growth of the foreign debt, governed solely by the international interest rate:

$$\frac{dD}{dt} = rD \tag{19}$$

If the behaviour of the interest rate is known, it is possible to study the trajectory of the indebtedness level of the economy, defined by the ratio of the foreign debt to income. Using equation (18), we write the income variable as:

$$Y(t) = Y(0)e^{\left(\frac{\delta}{\beta} g_f\right)t} \tag{20}$$

Solving equation (19), we obtain the expression that defines the foreign debt of the economy at a given point in time. For simplicity, assume the interest rate to be constant, so that:

$$D(t) = D(0)e^{rt} \tag{21}$$

In this scenario, foreign debt is, therefore, defined according to the following functional form:

$$\frac{D(t)}{Y(t)} = \left( \frac{D(0)}{Y(0)} \right) e^{\left(-\frac{\delta}{\beta} g_f\right)t} \tag{22}$$

According to equation (22), the level of debt at a given point in time, $t$, depends on the observed initial debt. However, its behaviour is determined fundamentally by the exponential term. If the international interest rate is greater than the growth rate of the economy, the debt presents an explosive behaviour, despite its initial value. Given the structural parameters of the economy and the growth rate of the world economy, the stability of the indebtedness depends exclusively on the behaviour of the interest rate. There is, thus, a clear state of vulnerability in face of foreign financial conditions. This vulnerability is exacerbated if the
interest rate is a growing function of the foreign debt itself, given the existence of a risk premium by international creditors. As an example, let us deal with a particular case in which:

\[ r_t(t) = r^* + \tau \frac{D_t}{Y_t} \]  

(23)

In this case the interest rate is specified as a linear function of debt, where \( \tau \) and \( r^* \) are parameters greater than zero. Given this specification, the differential equation describing the accumulation of foreign debt becomes non-linear. Solving the mentioned equation, we obtain the following expression for foreign debt:

\[ \frac{D_t}{Y_t} = \frac{e^{\left( r^* - \frac{\delta}{2} g_f \right) t}}{Y_0 + \frac{\tau}{D_0} - \frac{\tau}{r^* - \frac{\delta}{2} g_f} e^{\left( r^* - \frac{\delta}{2} g_f \right) t}} \]  

(24)

This equation shows that if the parameter \( r^* \), the interest rate, is higher than the growth rate of the economy, the exponential term, that appears in the numerator and in the denominator in the right side of the equation, grows with time. As a consequence, the denominator tends to zero in a finite time \( T \), thereby leading to an explosion of the debt. The exact moment of the collapse can be easily calculated, arriving at the following result:

\[ T = \frac{1}{r^* - \frac{\delta}{2} g_f} \ln \left[ \frac{Y_0 \left( r^* - \frac{\delta}{2} g_f \right)}{D_0 \tau} \right] \]  

(25)

According to equation (25), the higher the value of the parameter \( \tau \), which reflects the responsiveness of the interest rate to the increase in debt, the shorter is the time for the mentioned collapse.

**CASE (B)**

We now analyse a different context, in which the domestic economy absorbs a growing volume of speculative investment and interest payments are fully met at the due time. In this analysis, the inflow of speculative capital is, by hypothesis, a growing function of the world income. We describe this in the following functional form:

\[ \text{13 In other words, the indebtedness tends to } + \infty. \]

13
\[ I_e = k Y_f^e \]  

(26)

where \( k \) is a constant greater than zero. According to this function, the speculative investment grows exponentially with the world income, with an elasticity \( \varepsilon > 0 \) assumed constant.

Using this expression and considering that interest is entirely paid, equation (13) now shows:

\[
\frac{P_d}{E} b \left( \frac{P_d}{P_f E} \right)^\eta Y_f^\delta + k Y_f^e = P_f a \left( \frac{P_f E}{P_d} \right)^\psi Y^\beta + rD
\]

(27)

While the evolution of the external debt is described as:

\[
dD = k Y_f^e
\]

(28)

Assuming the growth rate of the world economy to be constant, the solution of the differential equation is:

\[
D(t) = D(0) + \frac{k}{\varepsilon g_f} Y_f^e(t)
\]

(29)

Substituting this result in equation (27), we obtain the following expression:

\[
\frac{P_d}{E} b Z^{-\eta} Y_f^\delta + KY_f^e = P_f a Z^\psi Y^\beta + r \left[ D(0) + \frac{k}{\varepsilon g_f} Y_f^e \right]
\]

(30)

Again, consider the interest rate to be constant in time. Given this hypothesis and after algebraic operations, equation (30) is re-written as:

\[
Y^\beta = J e^{(\xi f)} + L \left( 1 - \frac{r}{\varepsilon g_f} \right) e^{(\xi f)} - W
\]

(31)

where \( J, L \) and \( W \) are constants greater than zero.\(^1\) The analysis until now assumes that there is no initiative whatsoever of the domestic economic authorities in restricting the inflow of foreign capital. We shall formally introduce this possibility by defining a control variable, named \( \lambda \), which acts upon the elasticity of the speculative capital inflows relative to the growth of the world economy. More precisely, \( \lambda \) is a multiplicative factor of the parameter \( \varepsilon \), so that the product is close to zero in a regime of complete capital control and equal to \( \varepsilon \) when there is free mobility. Hence, \( 0 < \lambda \leq 1 \).

With no capital controls, equations (29) and (31) reveal at least three possible scenarios regarding the growth rate and the foreign debt of the economy over time. The first scenario is when

\(^1\) Where \( Z \) is the expression for the real exchange rate, assumed constant.

\(^1\) It should be mentioned that the expression is defined only for positive values of the income variable.
\[ \varepsilon > \delta \quad \text{and} \quad r < \varepsilon g_f \quad (B.1) \]

In a sufficiently long time span, we get

\[ Y^\beta \approx e^{(eg_f)k} \quad (32) \]

And, therefore:

\[ g \approx \frac{\varepsilon}{\beta} g_f > \frac{\delta}{\beta} g_f \quad (33) \]

Consequently, under these circumstances the economy tends to grow faster than what it is predicted by Thirlwall's Law.\(^\text{16}\) Nevertheless, it is important to notice what happens with the level of foreign debt. In the long-term,

\[ D \approx e^{(eg_f)k} \quad (34) \]

And the resulting expression for foreign debt becomes:

\[ \frac{D}{Y} \approx e^{\left(1-\frac{1}{\beta}\right)g_f k} \quad (35) \]

From this expression we conclude that if \( \beta \), the income elasticity of imports, is greater than one – the empirically most probable case, especially for developing countries – then debt is asymptotically explosive. We can thus infer that, under these conditions, accelerated growth would not be sustainable. The country would face serious risks of a currency crisis. Let us illustrate this scenario with a numeric example, doing a simulation exercise.\(^\text{17}\) Assume that \( \delta=1.5 \), \( \beta=2.5 \) and \( \varepsilon=2.25 \). The initial external debt level is 0.25. The world economy supposedly grows at a constant yearly rate of 4%, which is the same level of the international interest rate.\(^\text{18}\)

Under these conditions, the simulation shows that the economy displays, during a ten-year period, an average growth rate of 2.9%. This is, in fact, higher than the rate of Thirlwall's law, which is of 2.4%. Nonetheless, foreign debt sharply increases, as expected – it reaches approximately 0.5 in the end of the period (twice the initial amount), characterising a situation of high vulnerability. The same exercise was made setting a higher interest rate, of 6%; the growth of the economy was slightly lower, averaging 2.81% per year, while the final debt was slightly higher, reaching 0.52. The behaviour previously presented by both of these variables is maintained in this case. The charts below display the trajectory of debt over time for both situations:

\(^\text{16}\) Given a growth rate of the world economy greater than zero, a condition assumed during the analysis.
\(^\text{17}\) The simulations shown in this work were done with the software Free Pascal.
\(^\text{18}\) The values chose for the parameters and the initial conditions of the problem are based on the recent experience of the Brazilian economy, with reference to the 1990-2000 period. The relative magnitude of the income elasticity of exports and of imports was inferred based on the growth of the Brazilian economy relative to the world economy during the period. Research sources: IPEADATA; WDI (2005); Jayme Jr. (2004); Goldfajn e Minella (2005).
Alternatively, assume the following conditions:

\[ \varepsilon < \delta \quad \text{and} \quad \bar{r} < \varepsilon g_f \quad (B.2) \]

From equation (31), it follows that in this case the dominant exponential term (in the right side) is the one containing the parameter \( \delta \). Hence, in the long-term:

\[ Y^\beta \approx e^{(\delta g_f)} \quad (36) \]
The economy thus grows at approximately the rate stipulated by Thirlwall's model (1979). Regarding foreign debt, there are two diametrically opposed possibilities associated to the following result:

\[
D = e^{\left(\frac{\epsilon - \delta}{\beta}\right)g_f}
\]

If \( \epsilon \leq \frac{\delta}{\beta} \),
then the debt level decreases over time, asymptotically tending to zero.

On the other hand, if \( \epsilon > \frac{\delta}{\beta} \),
then the foreign debt of the economy grows over time, following an explosive pattern. A third scenario covered by case (B) occurs when

\[
\epsilon > \delta \quad \text{and} \quad \bar{r} > \epsilon g_f \quad (B.3)
\]

In this context, equation (31) suggests a negative growth of the domestic economy in the long-term. The income tends to zero and, consequently, foreign debt explodes.

In sum, the three scenarios point to a serious risk of explosive foreign debt. The analysis shows that, under the theoretical conditions considered, the access to speculative capital possibly does not lead to a long-term economic growth rate above that predicted by Thirlwall's Law. Actually, one of the possibilities is a growth rate below this level, which means tightening the foreign constraint. This perspective suggests that the use of instruments to restrict speculative capital can be favourable to the long-term growth of the economy, despite possible short-term reductions of the economic expansion.

We can now analyse the impact of the capital control variable, \( \lambda \). Initially, let the domestic economy face the conditions of scenario (B.1). Under these circumstances, we saw that foreign capital inflows initially lead to faster growth, but the foreign debt level (probably) displays an explosive trajectory, thus deterring the continuity of the process. The parameter \( \epsilon \), which defines the rhythm of speculative capital inflows for a given growth rate of the world economy, plays a central role in defining the different scenarios. If we multiply this parameter by the variable \( \lambda \), the result can possibly lead the economy to the conditions that define scenario (B.2). In the latter, the growth rate is the one predicted by Thirlwall's Law, but the external debt can decrease over time. Nevertheless, the result of the policy is conditioned by the structural domestic parameters, \( \delta \) and \( \beta \), and by the external conditions, represented by the growth of the world economy and the international interest rate.

The capital control policy will lead to the decreasing debt scenario under the following conditions:
\[ \lambda \epsilon < \frac{\delta}{\beta} \quad \text{and} \quad \bar{r} < \lambda \epsilon g_f \]

However, given the conditions of the problem this configuration may not be plausible, as shown before by the numerical exercise. We assumed the international interest rate and the growth rate of the world economy to be both equal to 4\%, and the ratio \( \delta/\beta \) equal to 0.6\%. With these values the second condition cannot be satisfied, whatever may be the result of \( \lambda \epsilon < \delta/\beta \). In spite of that, it can still be numerically shown that, even in this case, capital controls can, albeit limitedly, contribute to a lower increase of the debt – and, consequently, a reduction of the foreign vulnerability. This will happen if \( \lambda \epsilon < \delta \). Maintaining the same basic initial conditions assumed before, an example of a simulation with \( \lambda \epsilon = 1.25 \) leads to an average growth of 2.49\% and a final debt level, after ten years, of 0.46. With \( \lambda \epsilon = 0.75 \), the average growth is of 2.33\% per year and the final debt level 0.44. The foreign debt in the two regimes is described, respectively, by charts 3 and 4.

**Figure 3 – Debt**

![Figure 3](image)

**Figure 4 – Debt**

![Figure 4](image)
CASE (C)

Consider the case with nonzero foreign direct investment, assuming its evolution over time to be a function of the growth rate of the domestic economy:

\[ \frac{dI_d}{dt} = C(g - g_c) \quad (39) \]

where \( C \) is a parameter greater than zero. This equation shows that the direct investment grows over time if the growth rate of the economy is greater than a certain critical value, \( g_c \), which we assume to be known. Due to the formal complexity of this case, it will be analysed in strictly numerical terms, with the use of computer simulations. We initially assume there is no capital controls, so that \( \lambda \varepsilon = \varepsilon \).

Let us return to our initial example, with \( \delta = 1.5, \beta = 2.5 \) and \( \varepsilon = 2.25 \). The international interest rate and the growth rate of the world economy are equal to 4%. Assume initially that the direct investment and the speculative investment represent identical parcels of the total inflow of foreign capital.\(^\text{19}\) The critical value, \( g_c \), is initially equal to 2.40%.\(^\text{20}\) Let \( C \) be 5.6%.\(^\text{21}\) The simulation for this scenario displays an average growth rate of 2.66% per year. This value is slightly higher than the one foreseen by Thirlwall’s Law, while lower than what was obtained in the absence of direct investment. However, in comparison to this last case, the final foreign debt after the ten-year period is considerably lower, reaching 0.36 (as compared to 0.5).\(^\text{22}\)

\[ \lambda \varepsilon = \varepsilon. \]

---

\(^{19}\) The other initial conditions of the problem are kept unchanged.

\(^{20}\) Which is Thirlwall’s (1979) growth rate.

\(^{21}\) This value was determined so that, for a small divergence of the growth rate of the economy in relation to the critical value (circa 0.09%), the variation of the direct investment is of approximately 0.50, which corresponds to 10% of the total capital inflow originally calculated. Actually, the functional form assumed for the evolution of the direct investment guarantees that the long-term results show very little sensitivity to the value of the parameter \( C \).

\(^{22}\) A reduction of approximately 28%.
Moderately better numbers emerge when we reduce the critical value for the direct investment, considering $g_c=1.20\%$. In this case, the average growth rate of the economy is 2.74\% per year, and the debt level reaches 0.35.

These scenarios suggest, under the conditions of the model, that policies aiming direct investment in lieu of speculative investment may reduce the external vulnerability of the economy. They thus enable more favourable prospects of a sustainable economic growth. We must now analyse the impact of our capital control variable, $\lambda$, in this context.

Assume $g_c=2.40$ and $\lambda e=1.25$. In this situation, the average growth rate is 2.44\% per year (which is very close to what is predicted by Thirlwall's Law) and the foreign debt, at the end of the period, reaches 0.33. The results display, therefore, a slightly lower growth and smaller debt as compared to the situation with free capital mobility.

For $\lambda e=0.75$, the average growth and the final debt are, respectively, 2.35\% per year and 0.32. Another interesting result of the capital control policy is that the composition of the capital inflow is altered in favour of direct investment. The initial $I_d/I_c$ ratio was 1, by assumption. In the free capital mobility scenario, after five years this ratio was reduced to 0.58, reaching 0.40 after 10 years. In the scenario with $\lambda e=1.25$, these values were 0.72 and 0.50, while with $\lambda e=0.75$ they were 0.8 and 0.7. These numbers support the proposition that mechanisms for the control of capital inflows can help reduce the fragility of an economy facing difficulties in its balance-of-payments.

### Figure 6 – Debt

![Graph showing debt with $\lambda e=1.25$.](image)

5. Final remarks

In this paper, the balance-of-payments constrained growth conception we deal with here identifies a possible theoretical explanation, alternative to the neoclassical one, for the international disparity of the long-term growth of the countries. This explanation is strongly based on the structural aspects that determine the countries' structure of production and their divergent roles in international trade. This sort of analysis is particularly relevant for understanding the difficulties less developed economies face in overcoming their relative backwardness. Based on this conception, we developed an alternative analysis for approaching the relationship between economic growth and foreign debt, highlighting the
possible contribution of the control of capital inflows. This study leads to three main conclusions.

First, that the unrestricted access to international capital flows can increase the balance-of-payments constraint to growth, due to excessive foreign debt. The latter leads to increasing expenditures with interest payments and an ever-growing vulnerability of the domestic economy in face of the oscillations of the world economy, which can lead to a currency crisis. The second point is that capital controls can be desirable as a means of avoiding, or at least diminishing, the process of mounting fragility. When approaching this issue, it is definitely necessary to establish an analytical distinction between different types of capital flows in regard to their divergent characteristics and economic impact.

There is a third conclusion to be drawn from the analysis of the role of foreign investment in overcoming the balance-of-payments constraint of less developed economies. In order for this capital to really be effective, it must provide not only an immediate relief of the currency shortage, but also act on the structural aspects that ultimately determine the constraint imposed by the balance-of-payments on long-term growth. An important challenge, for future research, lies in integrating these theoretical elements in an empirical investigation. This is relevant due to the significance of the issue at hand and to the urgency of proposing development policies in an international context recurrently marked by financial instability.

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